ISSN 2229-5518

Mathematical Model for Several Category Organizations through Stochastic processes

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Abstract

n category organization subjected to random depart of personnel due to policy decisions taken by the organization is considered. we have found mathematical model and analytical results for the mean of the time for employment at any organization.

Key words: man hours, depart, breakdown point, employment, Mean.

1. Introduction

Maintenance activities are the backbone of a successful and profitable organization. Assuming that, organization has n categories of personnel and that the loss of man hours. If maximum loss of man hours due to the depart of personnel crosses a particular level, known as threshold for the organization, the organization reaches an uneconomic status which otherwise be called breakdown point and employment is to be done at this point. In this paper we construct the time for employment in n category organization. The breakdown point for the organization is the sum of the constant breakdown points for the loss of man hours.

2. Model description

Consider an organization having n categories which takes policy decisions at random epochs in the interval $[0, \infty)$. At every decision making epoch a random number of persons depart the organization. There is a associated loss of man hours to the organization if a person depart. The loss of man hours is a sequence of independent and identically distributed random variables. Each category has its own breakdown point. The loss of man hours, inter decision time process and breakdown points are statically independent. In this paper, an organization with n category is considered and the mathematical model constructed for mean time for employment.

3. Main result

The time of employment is given by

$$P[T \succ t] = \sum_{k=0}^{\infty} V_k(t) P[\sum_{i=1}^{k} R_i \prec X]$$

T – Continuous random variable denoting the time employment in the organization.

$V_k(t)$ - Probability that there are exactly k – decision epochs in (0, t]

$R_i -$ Continuous random variable denoting the amount of depart caused to the system due to the exit of man hours to the i[th] decision.

$$P[\sum_{i=1}^{k} R_i \prec X] = \int_0^\infty P[X \succ \sum_{i=1}^{k} R_i / \sum_{i=1}^{k} R_i = r] g_k(r) dr$$

X – Continuous random variable the threshold level for the organization

$$= \int_0^\infty g_k (1 - H(r)) dr$$

H(.) – the distribution function of X

$$= \sum_{i=m}^{n} C_i [\int_0^\infty g_k(r) e^{-i\theta r} (1 - {}^{n-i}C_1 e^{-\theta r} + {}^{n-i}C_2 e^{-2\theta r} - \ldots (-1)^{n-i} e^{(n-i)\theta r}) dr]$$

Using convolution theorem on Laplace transform

$$= \sum_{i=m}^{n} C_i [(\bar{g}(i\theta)^k - {}^{n-i}C_1 (\bar{g}((i+1)\theta)^k + \ldots (-1)^{n-i} (\bar{g}(n\theta))^k]$$

From renewal theory,

$$V_k(t) = F_k(t) - F_{K+1}(t)] \text{ with } F_0(t) = 1$$

$$P[T \succ t] = \sum_{k=0}^{\infty} (F_k(t) - F_{k+1}(t)) \sum_{i=m}^{n} C_i \, [(\bar{g}(i\theta)^k -$$

$$^{n-i}C_1(\bar{g}(i+1)\theta)^k + \ldots(-1)^{n-i}(\bar{g}(n\theta))^k]$$

$F(.)$ - Distribution of the inter decision times with density function f(.)

$F_k(.)$ - k fold convolution of F(.)

Since from [11]

$$P[T \succ t] = 1 - \sum_{i=m}^{n} {}^nC_1\{(1-g(i\theta))^k \sum_{k=1}^{\infty} F_k(t)(\bar{g}(i\theta))^{k-1} -$$

$$^{n-i}C_1(g(i\bar{+}1)\theta)^k \sum_{k=1}^{\infty} F_k(t)(g(i+1)\bar{\theta})^{k-1} + \ldots$$

$$.(-1)^{n-i}(1-\bar{g}(n\theta))\sum_{k=1}^{\infty} F_k(t)(\bar{g}(n\theta))^{k-1}$$

The distribution time of the time for employment is

$$L(t) = \sum_{i=m}^{n} {}^nC_1\{(1-g(i\theta))^k \sum_{k=1}^{\infty} \bar{F}_k(t)(\bar{g}(i\theta))^{k-1} -$$

$$^{n-i}C_1(g(i\bar{+}1)\theta)^k \sum_{k=1}^{\infty} F_k(t)(g(i+1)\bar{\theta})^{k-1} + \ldots$$

$$.(-1)^{n-i}(1-\bar{g}(n\theta))\sum_{k=1}^{\infty} F_k(t)(\bar{g}(n\theta))^{k-1}$$

L(.) – Cumulative distribution functions of T.

Differentiate the above with respect to t

$$l(t) = \sum_{i=m}^{n} {}^nC_1\{(1-g(i\theta))\sum_{k=1}^{\infty} \bar{f}_k(t)(g(i\bar{\theta}))^{k-1} -$$

$$^{n-i}C_1(1-(\bar{g}(i+1)\theta)^k \sum_{k=1}^{\infty} f_k(t)(\bar{g}(i+1)\theta)^{k-1} + \ldots$$

$$.(-1)^{n-i}(1\bar{-}g(n\theta))\sum_{k=1}^{\infty} f_k(t)(g(n\bar{\theta}))^{k-1}\}$$

$f_k(.)$ - k fold convolution of g(.)

$l(.)$ - Probability density function of T.

Take Laplace transform on both sides we get

$$\bar{l}(s) = \sum_{i=m}^{n} {}^nC_1\{(1-g(i\theta))\sum_{k=1}^{\infty}(\bar{f}(s))^k(\bar{g}(i\theta))^{k-1} -$$

$$^{n-i}C_1(1-(\bar{g}(i+1)\theta)^k \sum_{k=1}^{\infty} \bar{f}(s))^k(\bar{g}(i+1)\theta)^{k-1} + \ldots$$

$$.(-1)^{n-i}(1\bar{-}g(n\theta))\sum_{k=1}^{\infty}(\bar{f}(s))^k(\bar{g}(n\theta))^{k-1}\}$$

$\bar{f}(.) -$ Laplace transform of g(.)

Suppose f and g are exponential densities with parameter λ_1 *and* λ_2

$$\bar{f}(s) = \frac{\lambda_1}{(\lambda_1 + s)}, \bar{g}(s) = \frac{\lambda_2}{(\lambda_2 + s)}$$

$$E(T) = [-\frac{d}{ds}\bar{l}(s)]_{s=0}$$

E(T) – mean time for employment

The mean time for employment is

$$E(T) = \frac{1}{\lambda_1}\sum_{i=m}^{n} {}^nC_i\{\frac{\lambda_2 + i\theta}{i\theta} + {}^{n-i}C_1\frac{\lambda_2 + (i+1)\theta}{(i+1)\theta} + \ldots$$

$$+(-1)^{n-i}\frac{\lambda_2 + n\theta}{n\theta}\}$$

4. Conclusion

For n category man hour system, the mean for the time for employment is derived. In this paper we conclude that for average time for employment at any organization having n category man hour system.

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